Published by Institute of Physics Publishing for SISSA
Received: *December 20, 2005*
Accepted: *January 3, 2006*
Published: *January 13, 2006*

Dimension two condensates and the Polyakov loop above the deconfinement phase transition

Eugenio Megías, Enrique Ruiz Arriola and Lorenzo Luis Salcedo

Departamento de Física Moderna, Universidad de Granada
E-18071 Granada, Spain
E-mail: emegias@ugr.es, earriola@ugr.es, salcedo@ugr.es

Abstract: We show that recent available lattice data for the renormalized Polyakov loop above the deconfinement phase transition exhibit unequivocal inverse power temperature corrections driven by a dimension 2 gluon condensate. This simple ansatz provides a good overall description of the data throughout the deconfinement phase until near the critical temperature with just two parameters. One of the parameters is consistent with perturbation theory while a second, non perturbative, parameter provides a numerical value of the condensate which is close to existing zero and finite temperature determinations.

Keywords: Thermal Field Theory, Nonperturbative Effects, QCD, Lattice QCD.

http://jhep.sissa.it/archive/papers/jhep012006073/jhep012006073.pdf

JHEP01(2006)073

Contents

1. Introduction

The Polyakov loop plays a relevant theoretical role in QCD at finite temperature. It represents the propagator of a static test quark and therefore it is crucial in the understanding of the confinement-deconfinement crossover. In [1, 2] it was related to a heavy quark free energy so its vanishing in quenched QCD signals the confinement phase. As noted by 't Hooft [3], gluodynamics at finite temperature formulated using the imaginary time formalism, has an extra discrete global symmetry, in addition to usual gauge invariance. This symmetry is spontaneously broken above the deconfinement phase transition [4, 5]. The Polyakov loop, $L(T)$, is a natural order parameter for such phase transition; under periodic gauge transformations L is an invariant object but under a 't Hooft transformation it picks up a factor which is an element of the center of the gauge group. Effective field theories for the Polyakov loop have been proposed in [6]. (For a comprehensive review see e.g. ref. [7]).

The smooth Wilson loops, and in particular the Polyakov loop, are composite operators. Their perturbative renormalizability was discussed in [8 – 11], finding the remarkable result that they are multiplicatively renormalizable, without mixing with other operators. Soon afterwards, the perturbative evaluation of the Polyakov loop was addressed by Gava and Jengo [12] within dimensional regularization, to next-to-leading order (NLO). After including finite temperature vacuum polarization effects through Debye mass insertion, the leading order term turns out to be $\mathcal{O}(g^3)$ instead of the naively expected $\mathcal{O}(g^2)$. Their result implies that at high enough temperatures the renormalized Polyakov loop should approach unity from above, a consequence of the non trivial factor introduced by the renormalization. (The expectation value of the bare Polyakov loop vanishes in the continuum limit in any phase.) Not much progress has been achieved after this early result. At present there are no perturbative calculations of the expectation value of the Polyakov loop beyond NLO. As noted in [12] a direct calculation would have to confront the proliferation of Feynman diagrams due to infrared divergences [13]. A different approach, related to the dimensional reduction technique, is discussed below.

On the non perturbative side, the bare Polyakov loop has often been studied numerically within lattice gauge theory calculations, however, a reliable definition and calculation of the renormalized Polyakov loop has been achieved only recently. The method introduced in ref. [14] for quenched QCD obtains the Polyakov loop as a byproduct of the heavy quark-antiquark potential at finite temperature, obtained from the correlation between two Polyakov loops at different separations. Comparison with the zero temperature potential for small separations allows a quite precise determination of the quark selfenergy to be removed and so of the Polyakov loop. The renormalized Polyakov loop is larger than unity for temperatures at and above $3\,T_c$, in agreement with the perturbative expectation. The same technique has been applied to two flavor QCD in [15]. A direct lattice calculation of the Polyakov loop has also been reported in ref. [16] using a different approach. In this case a single Polyakov loop is used. Comparison of data taken at different temperatures allows to determine the renormalization factor to be applied to the bare result. The results of these two approaches agree approximately near the phase transition, but for temperatures above $1.3\,T_c$ the behaviors turn significantly different. The differences could be due to the effects of finite lattice spacing or to ambiguities in the renormalization prescription.

High temperatures probe kinematical regions which up to the manifest breaking of the Lorentz invariance correspond to large Euclidean momenta in the zero temperature quantum field theory. In dimensional regularization in the $\overline{\mathrm{MS}}$ scheme one finds that to a given temperature T there corresponds an Euclidean scale $\mu \sim 4\pi T$ [17], so that $T_c = 270\,\mathrm{MeV}$ means $\mu = 3\,\mathrm{GeV}$. In this regime one expects Operator Product Expansion (OPE) ideas to apply and more specifically, at not too high temperatures, condensates and power corrections should play a role. Actually, following some older proposals [18], phenomenological requirements [19], theoretical studies [20] and lattice analyses [21 − 23] there has recently been mounting evidence that the lowest condensate order BRST invariant condensate is of dimension 2. Such a condensate is generally non-local but in the Landau gauge becomes the local operator $\langle A^2_{\mu,a}\rangle$, with $A_{\mu,a}$ the gluon field. Also the $\langle A_0^2\rangle$ condensate appears as a parameter in the calculation of the pressure at finite temperature [24].

In this work we investigate the role of condensates on the expectation value of the Polyakov loop. The Polyakov loop is closely related to the thermal expectation value of $\mathrm{tr}(A_0^2)$ (the NLO perturbative result can be obtained in this way) and so condensate contributions to this quantity would have immediate impact on the Polyakov loop. Our motivation is best exposed by drawing an analogy with the zero temperature quark-antiquark potential in quenched QCD. The potential is, of course, closely related to the correlation function of two thermal Wilson lines. The perturbative regime of the potential $V(r)$ corresponds to small separations, where the potential is approximately Coulombian. At separations of the order of $1/\Lambda_{\rm QCD}$ (there is no other scale in gluodynamics) a linearly confining term develops and starts becoming dominant. Both pieces of the potential evolve under the renormalization group at a logarithmically slow rate. Therefore, modulo radiative corrections, the dimensionless quantity $rV(r)$ is composed of a flat perturbative piece plus a power-like term of the type $\Lambda_{\rm QCD}^2 r^2$ which is non perturbative. In analogy, at high temperatures, we can consider the behavior of the dimensionless quantity $\langle {\rm tr}(A_0^2)\rangle/T^2$, also directly related to the correlation function of two thermal Wilson lines. The analogous of the scale r in the previous case is the scale $1/T$ here, and certainly for large T the quantity $\langle {\rm tr}(A_0^2)\rangle/T^2$ is perturbative and flat modulo a logarithmic dependence. At lower temperatures we contemplate the possibility of non perturbative power-like terms of the type $\Lambda_{\rm QCD}^2/T^2$ to develop. As we show in this work, such term enters naturally through OPE corrections to the gluon propagator driven by condensates. An analysis of available lattice data turns out to display precisely the power-like pattern expected from the previous considerations. The pattern is followed in the deconfinement phase from the highest temperatures available down to near to the transition where deviations start to show up.

The paper is organized as follows. In section 2 we discuss perturbative aspects of the Polyakov loop and the use of dimensional reduction to attempt the calculation beyond NLO. In section 3 we show that the presence of condensates introduce a power-like pattern in the logarithm of the Polyakov loop expectation value. In section 4 we analyze the lattice data and show that they are fairly well described as a composition of perturbative plus condensate contributions. Finally, in section 5 we summarize our conclusions.

2. The perturbative Polyakov loop

2.1 Perturbative results

The (expectation value of the) Polyakov loop is defined as

$$L(T) = \left\langle \frac{1}{N_c} {\rm tr}\, \mathbf{P} \left(e^{ig\int_0^{1/T} dx_0 A_0(\boldsymbol{x},x_0)} \right) \right\rangle \tag{2.1}$$

where $\langle\ \rangle$ denotes vacuum expectation value, tr is the (fundamental) color trace, and $\mathbf{P}$ denotes path ordering. A_0 is the gluon field in the (Euclidean) time direction, $A_0 = \sum T_a A_{0,a}$, T_a being the Hermitian generators of the SU(N_c) Lie algebra in the fundamental representation, with the standard normalization ${\rm tr}(T_a T_b) = \delta_{ab}/2$.

As a composite operator the Polyakov loop is subject to renormalization. The multiplicative renormalizability of the Polyakov loop was established in refs. [8 − 11] in the context of perturbation theory. Gava and Jengo [12] addressed the perturbative computation of $L(T)$ in pure gluodynamics. The calculation was carried out to NLO, which corresponds to $\mathcal{O}(g^4)$, using dimensional regularization and in the Landau gauge. The result is of course gauge invariant. Explicitly,

$$L(T) = 1 + \frac{1}{16\pi}\frac{N_c^2-1}{N_c} g^2 \frac{m_D}{T} + \frac{N_c^2-1}{32\pi^2} g^4 \left(\log\frac{m_D}{2T} + \frac{3}{4} \right) + \mathcal{O}(g^5)\,. \tag{2.2}$$

Here m_D is the Debye mass, which controls the screening of chromoelectric modes in the plasma. To one loop [25]

$$m_D = gT(N_c/3 + N_f/6)^{1/2} \,, \tag{2.3}$$

N_c being the number of colors and N_f the number of flavors, to account for dynamical quarks. The coupling constant g runs with the temperature following the standard renormalization group analysis and one expects (2.2) to hold for high enough temperature. Remarkably, $L(T)$ turns out to be larger than unity implying that the renormalized Polyakov loop is not a unimodular matrix. Note that m_D contains a g and so the first non trivial contribution to L is $\mathcal{O}(g^3)$, due to the infrared structure of the theory, rather than the naively expected $\mathcal{O}(g^2)$. Note also that the perturbative result (as well as m_D) has a well defined large N_c limit, with 't Hooft prescription of keeping $g^2 N_c$ fixed.

2.2 Dimensional reduction

The result just quoted is rather old yet no higher order computations are presently available. Most efforts in perturbative high temperature QCD have been addressed to obtain the pressure and only recently such computations have been taken to their highest possible perturbative order [26], using dimensional reduction ideas [27, 28, 25, 29, 30]. In order to subsequently include possible contributions from condensates, we will presently reproduce the lowest order perturbative result for $L(T)$ using the dimensional reduction approach. In addition this will allow us to discuss properties of higher order perturbative contributions to $L(T)$.

The starting point is the Euclidean QCD action ($D_\mu = \partial_\mu - ig_0 A_\mu$, $F_{\mu\nu} = ig_0^{-1}[D_\mu, D_\nu]$, N_f massless fermions)

$$\mathcal{L}_{\rm QCD} = \frac{1}{2}\,{\rm tr}(F_{\mu\nu}^2) + \bar\psi \, \not{D} \, \psi + \mathcal{L}_{\rm gf+gh+ct} \,, \tag{2.4}$$

where $\mathcal{L}_{\rm gf+gh+ct}$ accounts for gauge fixing and ghost terms as well as the counterterms for renormalization. Next, one proceeds to integrate out the fermionic modes and all non stationary gluon modes, which become very heavy at high temperature. This results in an effective theory for the remaining stationary (time-independent) gluon modes $A_\mu(\boldsymbol{x})$, described by a three dimensional action $\int d^3x\, \mathcal{L}_3(\boldsymbol{x})$. To one loop and in the Landau gauge one obtains [17, 31, 30, 32]

$$\begin{aligned} T\mathcal{L}_3(\boldsymbol{x}) = {} & m_D^2\,{\rm tr}(A_0^2) + \frac{g^4(\mu)}{4\pi^2}({\rm tr}(A_0^2))^2 + \frac{g^4(\mu)}{12\pi^2}(N_c - N_f)\,{\rm tr}(A_0^4) \\ & + \frac{g^2(\mu)}{g_E^2(T)}\,{\rm tr}([D_i, A_0]^2) + \frac{g^2(\mu)}{g_M^2(T)}\frac{1}{2}\,{\rm tr}(F_{ij}^2) + T\delta\mathcal{L}_3 \end{aligned} \tag{2.5}$$

where $g(\mu)$ is the running coupling constant in the $\overline{\rm MS}$ scheme (to be used in the Debye mass and in the Polyakov loop formula too)

$$\frac{1}{g^2(\mu)} = 2\beta_0 \log(\mu/\Lambda_{\overline{\rm MS}}) \,, \qquad \beta_0 = (11N_c/3 - 2N_f/3)/(4\pi)^2 \tag{2.6}$$

and

$$\begin{aligned}
\frac{1}{g_E^2(T)} &= \frac{1}{g^2(\mu)} - 2\beta_0(\log(\mu/4\pi T)+\gamma_E) + \frac{1}{3(4\pi)^2}\left(N_c + 8N_f\left(\log 2 - 1/4\right)\right) \,, \\
\frac{1}{g_M^2(T)} &= \frac{1}{g^2(\mu)} - 2\beta_0(\log(\mu/4\pi T)+\gamma_E) + \frac{1}{3(4\pi)^2}\left(-N_c + 8N_f\log 2\right) \,.
\end{aligned} \tag{2.7}$$

The remainder $\delta\mathcal{L}_3$ contains operators of mass dimension 6 and higher. In addition there are higher loop terms and constant (field independent) terms which would be relevant for the pressure. (Note that g_E and g_M are not to be confused with the coupling constants under the same name appearing, e.g., in [33].)

At lowest order we will only need the mass term and the kinetic energy term of the chromoelectric field (first and fourth terms respectively in eq. (2.5)). It will be convenient to work with a rescaled A_0 field equal to $g(\mu)/g_E(T)$ times the $\overline{\mathrm{MS}}$ A_0 field. To all effects, including the Debye mass and the Polyakov loop formula which depends on the product gA_0, this is equivalent to using the new A_0 field together with $g_E(T)$ as coupling constant. The latter will be denoted $g(T)$ or just g from now on,

$$\begin{aligned}
\mathcal{L}_3(\boldsymbol{x}) &= \frac{m_D^2}{T}\,\mathrm{tr}(A_0^2) + \frac{1}{T}\,\mathrm{tr}([D_i, A_0]^2) + \cdots \,, \\
\frac{1}{g^2(T)} &= 2\beta_0\log(T/\Lambda_E)\,,
\end{aligned} \tag{2.8}$$

with

$$\Lambda_E = \frac{\Lambda_{\overline{\mathrm{MS}}}}{4\pi}\exp\left(\gamma_E - \frac{N_c + 8N_f(\log 2 - 1/4)}{22N_c - 4N_f}\right) \,. \tag{2.9}$$

For computing the QCD pressure one can use any gauge fixing to integrate the non stationary modes. This is an intermediate step to carry out the integration of the remaining modes. Consequently covariant gauges are often used as they are computationally simpler. For the Polyakov loop computation the situation is different; static gauges are preferred to covariant ones [25]. A static gauge is one in which $A_0(x)$ is brought to be time independent by means of a suitable gauge transformation. In such a gauge eq. (2.1) becomes

$$L = \frac{1}{N_c}\left\langle \mathrm{tr}\; e^{igA_0(\boldsymbol{x})/T}\right\rangle \,. \tag{2.10}$$

i.e., L depends only on the stationary mode of A_0 and so no information is lost on the Polyakov loop operator if the non stationary modes are integrated out. Unfortunately, the necessary perturbative computations of e.g. $\mathcal{L}_3(\boldsymbol{x})$, are only available for covariant gauges. Only in a static gauge the stationary mode $A_0(\boldsymbol{x})$ coincides with the logarithm of the Polyakov loop operator. Therefore, in a covariant gauge the effective action of the stationary mode is insufficient to recover Polyakov loop expectation values[1]. Nevertheless, as we discuss below, the gauge dependence only affects beyond NLO and the two coefficients in eq. (2.2) are reproduced using the formulas in, for instance, [33, 26] and the method explained in the next subsection.

[1] Using the stationary mode in eq. (2.10) amounts to removing the path ordering operator in the definition of the Polyakov loop, rendering it a gauge dependent quantity.

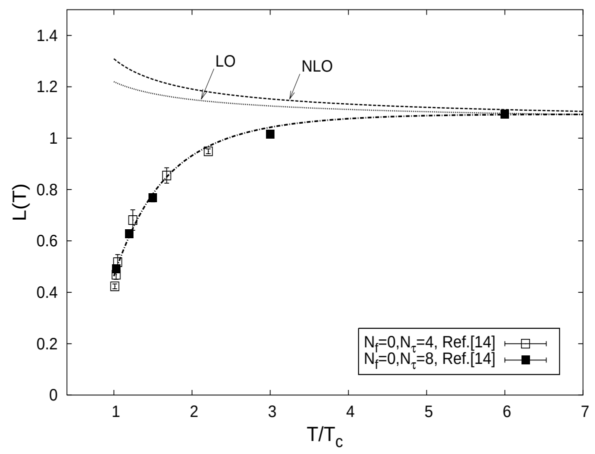

Figure 1: The renormalized Polyakov loop versus the temperature, in gluodynamics. Lattice data from [14]. Perturbative LO and NLO results are shown for comparison. The curve follows from a fit of the parameter b in eq. (4.4).

Doing a series expansion of $L(T)$ in eq. (2.10) one gets

$$L(T) = 1 - \frac{g^2}{2T^2}\frac{1}{N_c}\langle \mathrm{tr}(A_0^2)\rangle + \frac{g^4}{24T^4}\frac{1}{N_c}\langle \mathrm{tr}(A_0^4)\rangle + \cdots . \tag{2.11}$$

$\mathrm{tr}(A_0)$ vanishes identically while the other terms of odd order are assumed to vanish due to the QCD conjugation symmetry, $A_\mu(x) \to -A_\mu^T(x)$. The leading contribution is then attached to $\langle \mathrm{tr}(A_0^2)\rangle$. This quantity has dimensions of mass squared and so it would vanish in a perturbative calculation at zero temperature. At finite temperature instead it should scale as T^2 modulo slowly varying radiative corrections. Let $D_{00}(\boldsymbol{k})\delta_{ab}$ denote the momentum space propagator for the canonically normalized fields $T^{-1/2}A_{0,a}(\boldsymbol{x})$, then

$$\langle A_{0,a}^2\rangle = (N_c^2-1)T\int\frac{d^3k}{(2\pi)^3}D_{00}(\boldsymbol{k})\,. \tag{2.12}$$

To lowest order the three dimensional propagator is

$$D_{00}^{\mathrm{Pert}}(\boldsymbol{k}) = \frac{1}{\boldsymbol{k}^2+m_D^2}\,, \tag{2.13}$$

where the upperscript Pert indicates that it is a perturbative contribution. When this is inserted in (2.12) it yields (we apply dimensional regularization rules)

$$\langle A_{0,a}^2\rangle^{\mathrm{Pert}} = -(N_c^2-1)\frac{Tm_D}{4\pi}\,. \tag{2.14}$$

This result used in eq. (2.11) (and using $\mathrm{tr}(A_0^2) = A_{0,a}^2/2$) reproduces the perturbative value of $L(T)$ to $\mathcal{O}(g^3)$.

In figure 1 we compare the perturbative $L(T)$ in eq. (2.2) with a recent lattice determination of this quantity in pure gluodynamics and $N_c = 3$ [14]. As we can see, in

JHEP01(2006)073

the high temperature region, T about $6\,T_c$, the $L(T)$-lattice is larger than unity, as predicted by the perturbative calculation, moreover the numerical value is also consistent with perturbation theory. The agreement quickly deteriorates as the critical temperature is approached from above; while the lattice data moves downwards, to eventually displaying a phase transition, the perturbative curve increases slightly. As expected, the perturbative result is slowly varying with temperature, the variation coming from logarithmic radiative corrections.

2.3 Higher perturbative orders

Let us now discuss higher order perturbative contributions to $L(T)$. The renormalizable pieces of the three dimensional Lagrangian are of the form

$$\mathcal{L}_3^{\rm ren} = \frac{1}{2}\,{\rm tr}(F_{ij}^2) + {\rm tr}([D_i,\mathcal{A}_0]^2) + m^2\,{\rm tr}(\mathcal{A}_0^2) + \lambda_1({\rm tr}(\mathcal{A}_0^2))^2 + \lambda_2\,{\rm tr}(\mathcal{A}_0^4) \tag{2.15}$$

with $\mathcal{A}_0 \sim T^{-1/2}A_0$, $m \sim gT$, and $\lambda_1 \sim \lambda_2 \sim g^4 T$. In addition, $D_i = \partial_i - ig_3\mathcal{A}_i$ with $\mathcal{A}_i \sim T^{-1/2}A_i$ and $g_3 \sim T^{1/2}g$. For $N_c = 2$ or $N_c = 3$ the λ_2 term is redundant and one can set $\lambda_2 = 0$. The vacuum energy density of this theory, $f(g_3, m, \lambda_1)$, has been computed to four loops in [26], with g_3, m and λ_1 as independent parameters. This allows to compute $\langle A_0^2\rangle$ and $\langle A_0^4\rangle$ by taking derivatives of f with respect to m^2 and λ_1 respectively, to obtain a perturbative estimate of the Polyakov loop. The general structure of the vacuum energy density is as follows [26]

$$f(g_3, m, \lambda_1) = \sum_{\ell\geq 1}\sum_{k=0}^{\ell-1} f_{\ell k}\, m^{4-\ell} g_3^{2k}\lambda_1^{\ell-k-1} \tag{2.16}$$

where ℓ denotes the number of loops and the coefficients $f_{\ell k}$ depend logarithmically on m. Consequently, for the quantities in the expansion of $L(T)$ one finds

$$\begin{aligned} \frac{g^2}{T^2}\langle {\rm tr}(A_0^2)\rangle &\sim \frac{g^2}{T}\frac{\partial f(g_3,m,\lambda_1)}{\partial m^2} \sim \sum_{\ell\geq 1}\sum_{n=\ell+2}^{3\ell} g^n\,, \\ \frac{g^4}{T^4}\langle {\rm tr}(A_0^4)\rangle &\sim \frac{g^4}{T^2}\frac{\partial f(g_3,m,\lambda_1)}{\partial \lambda_1} \sim \sum_{\ell\geq 2}\sum_{n=\ell+4}^{3\ell} g^n\,. \end{aligned} \tag{2.17}$$

As can be seen from these formulas, the first missing contribution to $L(T)$ would be $\mathcal{O}(g^7)$ from $\ell = 5$ in the $\langle {\rm tr}(A_0^2)\rangle$ term. The lowest contribution from $\langle {\rm tr}(A_0^4)\rangle$ at 5 loops is $\mathcal{O}(g^9)$ and that from $\langle {\rm tr}(A_0^6)\rangle$, not available from the computation, would first start at $\mathcal{O}(g^9)$ at 3 loops. So in principle, one could extend the perturbative result for $L(T)$ to $\mathcal{O}(g^6)$. Unfortunately, the matching relations which connect m, g_3 and λ_1 to the four dimensional QCD parameters are only available in covariant gauges for which the relation (2.10) does not apply. In particular, the ratio $g(\mu)/g_E(T)$ used above is gauge dependent at $\mathcal{O}(g^2)$ from two loop contributions, this would introduce a gauge dependence at $\mathcal{O}(g^5)$ in $L(T)$.

On the other hand, the non renormalizable terms $\delta\mathcal{L}_3$ ought to be examined as well to determine to which perturbative order they start contributing to $L(T)$. The leading such terms are schematically of the type [31, 32]

$$\delta\mathcal{L}_3 = \frac{g^2}{T^2}\,\mathrm{tr}([D_i,F_{\mu\nu}]^2) + \frac{g^3}{T^{3/2}}\,\mathrm{tr}(F_{\mu\nu}^3) + \frac{g^4}{T}\,\mathrm{tr}(A_0^2F_{\mu\nu}^2)\,. \tag{2.18}$$

Using the effective relation $D_i \sim gT$, the first term amounts to an $\mathcal{O}(g^4)$ correction to the kinetic energy, so it starts contributing at $\mathcal{O}(g^7)$ as a correction to the LO. The other terms are effectively of higher order.

Numerically the terms $\mathcal{O}(g^5)+\mathcal{O}(g^6)$ computed with the available matching relations do not make a substantial contribution as they are qualitatively and also quantitatively similar to those in [12]. Again the radiative nature of these perturbative terms produces a rather flat logarithmic dependence with the temperature in sharp contrast with the lattice data at not too high temperatures. This reinforces the need of non perturbative effects.

2.4 Gaussian ansatz

It is noteworthy that the contribution from $\langle A_0^4\rangle$ starts at $\mathcal{O}(g^6)$, and so to $\mathcal{O}(g^5)$ A_0 obeys a Gaussian distribution. That is, to this order one can replace (2.10) with

$$L = \exp\left[-\frac{g^2\langle A_{0,a}^2\rangle}{4N_cT^2}\right] \tag{2.19}$$

and so

$$\langle A_{0,a}^2\rangle^{\mathrm{Pert}} = -\frac{N_c^2-1}{4\pi}m_DT - \frac{N_c(N_c^2-1)}{8\pi^2}g^2T^2\left(\log\frac{m_D}{2T}+\frac{3}{4}\right) + \mathcal{O}(g^3)\,. \tag{2.20}$$

This formula holds also in the unquenched theory, since to this order N_f only appears through the Debye mass.

The Gaussian ansatz becomes correct $\mathcal{O}(g^5)$ at high enough temperature where the theory becomes weakly interacting due to asymptotic freedom. Also, it becomes exact in the large N_c limit as higher order connected expectation values are suppressed by powers of $1/N_c$. Note that $A_{0,a}^2$ scales as (N_c^2-1) and so L has a well defined limit with the standard prescription of keeping g^2N_c finite as $N_c\to\infty$. A Gaussian distribution for the Polyakov loop has been observed in lattice calculations [34]. The Gaussian ansatz is in fact equivalent to expanding the exponential, averaging over color degrees of freedom and finally invoking the vacuum saturation hypothesis ($\langle A_0^{2k}\rangle = (2k-1)!!\langle A_0^2\rangle^k$) routinely applied in QCD sum rules at zero temperature. In this line, the Wilson loop was discussed in ref. [35] by using the standard dimension 4 gluon condensate yielding for small contours a term proportional to the area squared of the contour. The situation has been revisited in ref. [36] in the context of dimension 2 condensates yielding an area law for small contours. This agrees with the observation in ref. [19] that dimension 2 condensates, effectively would-be tachyonic gluon masses, provide the short range signature of long range confining forces.

3. Condensate contributions to the Polyakov loop

As shown in figure 1 the perturbative contributions to the Polyakov loop expectation value describe only the region of very high temperature. This situation is reminiscent of what

JHEP01(2006)073

happens for the heavy quark-antiquark potential in QCD at zero temperature, as a function of the quark-antiquark separation. There, perturbation theory describes well the short distance region, where the theory is weakly interacting and standard one-gluon exchange produces a Coulomb-like potential. At larger distances confinement sets in and a linear potential must be added to account for the lattice data [37]. As the potential has dimensions of mass, the Coulomb piece does not need a dimensionful coefficient. This makes it allowable in perturbation theory, where $\Lambda_{\rm QCD}$ can only appear through logarithmic radiative corrections, as in eq. (2.6). On the other hand, the linear confining piece of the potential requires a dimension two coefficient, the string tension, which in pure gluodynamics should be $\Lambda^2_{\rm QCD}$ times a numerical coefficient. At one loop this implies a dependence $\exp(-1/\beta_0 g^2(\mu))$, the scale μ being related to the quark-antiquark separation r. While such contributions are perfectly possible in QCD, they are clearly beyond any finite order in perturbative QCD and can only be attained through suitable resummations of the perturbative series (see e.g. [38, 39]). It is noteworthy that the non perturbative dependence on g is not completely arbitrary, namely, it is such that $\Lambda_{\rm QCD}$ appears raised to positive integer powers. This finds a natural explanation from the OPE approach, where the non perturbative contributions are driven by condensates of concrete local operators. By dimensional counting, the condensate contributions carry a corresponding negative power momentum dependence, so they are subdominant at high momentum as compared to the purely perturbative terms but become more important at lower momenta, the lower dimensional operators being the dominant ones. In this line the confining piece of the zero temperature heavy quark-antiquark potential has been addressed phenomenologically by considering the contribution to the gluon propagator of a dimension two condensate, namely, $\langle A^2_\mu \rangle$ in the Landau gauge [18]. Just by dimensional counting such term produces a linearly confining term in the potential [19].

In this work we want to investigate the effect of low dimensional condensates on the Polyakov loop expectation value. The region of high temperatures is weakly interacting and so ideas inspired on the high momentum region of the zero temperature theory might be useful here. As shown above, at high temperatures, the Polyakov loop is closely related to the expectation value of A_0^2 in a static gauge. Perturbatively, such quantity necessarily scales as T^2, but non perturbatively a further term proportional to $\Lambda^2_{\rm QCD}$ is allowed. In order to account for non perturbative contributions coming from condensates, we will consider adding to the propagator new phenomenological pieces driven by positive mass dimension parameters. Specifically, we consider

$$D_{00}(\boldsymbol{k}) = D_{00}^{\rm Pert}(\boldsymbol{k}) + D_{00}^{\rm NonPert}(\boldsymbol{k}) \tag{3.1}$$

with the non perturbative term

$$D_{00}^{\rm NonPert}(\boldsymbol{k}) = \frac{m_G^2}{(\boldsymbol{k}^2 + m_D^2)^2} \,. \tag{3.2}$$

Such ansatz parallels those made at zero temperature in the presence of condensates [18, 19].

This new piece produces a non perturbative contribution to $\langle A_0^2\rangle$, namely,

$$\langle A_{0,a}^2\rangle^{\rm NonPert} = \frac{(N_c^2-1)Tm_G^2}{8\pi m_D}\,. \tag{3.3}$$

If we assume that m_G is temperature independent up to radiative corrections, the condensate will also be temperature independent, modulo these corrections. Equivalently, in terms of the condensate

$$D_{00}^{\rm NonPert}(\boldsymbol{k}) = \frac{8\pi}{N_c^2-1}\frac{m_D}{T}\frac{\langle A_{0,a}^2\rangle^{\rm NonPert}}{(\boldsymbol{k}^2+m_D^2)^2}\,. \tag{3.4}$$

Note that a positive condensate $\langle A_{0,a}^2\rangle^{\rm NonPert}$ indicates a would-be tachyonic gluon mass $-m_G^2$, as in [19].

Adding the two contributions to $\langle A_{0,a}^2\rangle$ in eq. (2.19), one obtains

$$-2\log L = \frac{g^2\langle A_{0,a}^2\rangle^{\rm Pert}}{2N_cT^2} + \frac{g^2\langle A_{0,a}^2\rangle^{\rm NonPert}}{2N_cT^2}\,. \tag{3.5}$$

The fact that, modulo radiative corrections (including running of the coupling and anomalous dimensions), $\langle A_{0,a}^2\rangle^{\rm Pert}$ scales as T^2 while $\langle A_{0,a}^2\rangle^{\rm NonPert}$ is temperature independent, suggests rewriting the previous formula as [40]

$$-2\log L = a + b\left(\frac{T_c}{T}\right)^2 \tag{3.6}$$

where the parameters a and b are expected to have only a weak temperature dependence. As advertised the non perturbative piece introduces a power-like dependence in the temperature which is not present in the perturbative calculation.

4. Comparison with lattice data

4.1 Results in gluodynamics

A reliable determination of the renormalized Polyakov loop in lattice gauge theory has been undertaken only recently in ref. [14], for pure gluodynamics and $N_c = 3$. This calculation is, of course, fully non perturbative. These authors compute the finite temperature correlation function of a heavy quark-antiquark pair for different separations. The two Polyakov loops are multiplicatively renormalized by extracting the (temperature dependent but separation independent) quark self energy in such a way that at short distances the standard zero temperature quark-antiquark potential is reproduced. At large separations the (squared) renormalized Polyakov loop is then obtained. That is, if P_x denotes the renormalized Polyakov loop operator located at x,

$$\langle P_xP_y\rangle = e^{-c(T)}\langle P_x^{\rm bare}P_y^{\rm bare}\rangle = e^{-F_{\bar qq}(r,T)/T} \underset{r\to\infty}{\longrightarrow} L^2(T)\,. \tag{4.1}$$

Motivated by the pattern in eq. (3.6), the lattice data for $-2\log L(T)$ are displayed versus $(T_c/T)^2$ in figure 2. As we can see the lattice data follow a nearly straight line. This

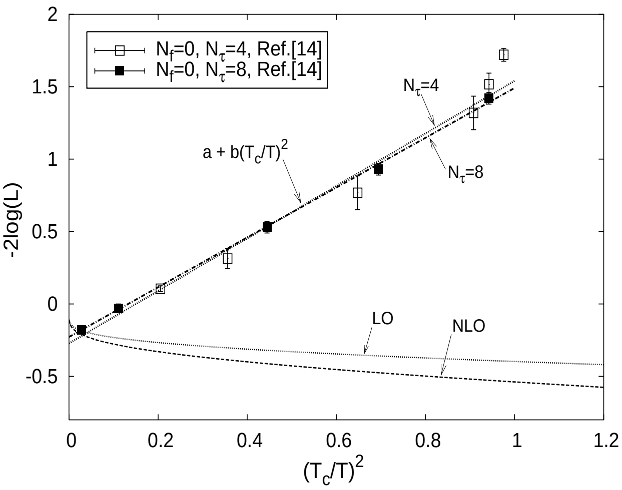

Figure 2: The logarithmic dependence of the renormalized Polyakov loop in gluodynamics versus the inverse temperature squared in units of the critical temperature. Lattice data from [14]. The fits use eq. (3.6) with a and b adjustable constants and lattice data above $1.03\,T_c$ for $N_\tau = 4$ and $N_\tau = 8$. Purely perturbative LO and NLO results for $N_f = 0$ are shown for comparison.

pattern is clearly distinguishable from the much flatter dependence predicted by the perturbative calculation, and unequivocally shows a temperature power correction characteristic of a dimension 2 condensate.

Identification of (3.6) with the formula (3.5) yields the relations

$$a = -\frac{1}{8\pi}\frac{N_c^2-1}{N_c}g^2\frac{m_D}{T} - \frac{N_c^2-1}{16\pi^2}g^4\left(\log\frac{m_D}{2T}+\frac{3}{4}\right) + \mathcal{O}(g^5)\,, \tag{4.2}$$

$$g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}} = 2N_cT_c^2 b\,. \tag{4.3}$$

A fit of the lattice data of the form

$$-2\log L = a^{\mathrm{NLO}} + b\left(\frac{T_c}{T}\right)^2 \tag{4.4}$$

with the perturbative value of a to NLO and b as a free constant parameter, yields

$$b = \begin{cases} 2.20(6)\,, \\ 2.14(4)\,, \end{cases} \qquad \chi^2/\mathrm{DOF} = \begin{cases} 0.75\,, & N_\tau = 4\,, \\ 1.43\,, & N_\tau = 8\,. \end{cases} \tag{4.5}$$

This corresponds to the following value for the condensate

$$g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}} = \begin{cases} (0.98 \pm 0.02\,\mathrm{GeV})^2\,, & N_\tau = 4\,, \\ (0.97 \pm 0.01\,\mathrm{GeV})^2\,, & N_\tau = 8\,. \end{cases} \tag{4.6}$$

In the fit we include lattice data for temperatures $1.03\,T_c$ or above. We use $T_c/\Lambda_{\overline{\mathrm{MS}}} = 1.14(4)$ [41, 37], and $T_c = 270(2)$ MeV [41]. Throughout this section we use the running coupling constant obtained from the beta function to three loops and Λ_E in eq. (2.9) as

JHEP01(2006)073

scale parameter. Assuming that the difference between the two lattice results is entirely due to finite cutoff effects, and assuming further that the corresponding leading effect goes as $1/N_\tau$, we obtain the estimate $(0.95(4)\,\mathrm{GeV})^2$ for $g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}}$ in the continuum limit.

We have also considered a fit of the lattice data with both a and b treated as free constant parameters. This produces

$$a = \begin{cases} -0.27(5)\,, \\ -0.23(1)\,, \end{cases} \qquad b = \begin{cases} 1.81(13)\,, \\ 1.72(5)\,, \end{cases} \qquad \chi^2/\mathrm{DOF} = \begin{cases} 1.07\,, & N_\tau = 4\,, \\ 0.45\,, & N_\tau = 8\,. \end{cases} \tag{4.7}$$

The values of χ^2/DOF are slightly better than the NLO prediction of a. Obviously the identification of a with the perturbative result will work better at high temperatures. Using eq. (4.2) we obtain for the highest temperature $6\,T_c$

$$a^{\mathrm{NLO}} = -0.22(1) \qquad (T = 6\,T_c)\,, \tag{4.8}$$

in qualitative agreement with the fitted values. Note that for this temperature the non perturbative power correction does contribute at the few percent level. For lower temperatures the NLO perturbative result evolves faster than the fit suggests. At this level of accuracy one should also take into account logarithmic corrections to the value of the condensate and eventually some anomalous dimension correction to the condensate. The present data do not allow a clean extraction of such fine details. The average value we get for the condensate with constant a is

$$g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}} = \begin{cases} (0.89 \pm 0.03\,\mathrm{GeV})^2\,, & N_\tau = 4\,, \\ (0.87 \pm 0.02\,\mathrm{GeV})^2\,, & N_\tau = 8\,, \end{cases} \tag{4.9}$$

a little lower than before. The corresponding continuum limit estimate results in $g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}} = (0.84(6)\,\mathrm{GeV})^2$.

We have attempted to determine the coefficient of a possible $1/T^4$ correction, appending formula (3.6) with a term $c(T_c/T)^4$. When we fit the lattice data for $N_\tau = 8$, this results in

$$b = 2.18(20)\,, \quad c = -0.04 \pm 0.24\,, \tag{4.10}$$

with $\chi^2/\mathrm{DOF} = 1.89$, where we have considered the perturbative value of a to NLO, and

$$a = -0.22(2)\,, \quad b = 1.61(24)\,, \quad c = 0.13 \pm 0.28\,, \tag{4.11}$$

with $\chi^2/\mathrm{DOF} = 0.42$, if we treat a as a free constant. The value of c is compatible with zero in any case, and the errors overlap with central values for a and b of eqs. (4.5) and (4.7) respectively. More accurate data are desirable in order to identify contributions from condensates of dimension 4.

It is noteworthy that a fit to the data completely excludes the existence of a term of the form $1/T$ in $\log(L(T))$. Such term would not have a theoretical basis, as no dimension one condensate exists. However, as noted by the authors of [14], there is a ambiguity in their procedure, which corresponds to adding a constant to the zero temperature quark-antiquark potential. Such ambiguity translates into an additive ambiguity in $F_{\bar{q}q}(r,T)$ in eq. (4.1), which would give rise a term of the type $1/T$ in $\log(L(T))$. The absence of such term indicates a preference for the Cornell prescription adopted in [14], namely, in $V_{\bar{q}q}(r) \sim v_0/r + v_1 + v_2 r$ to choose $v_1 = 0$ [42].

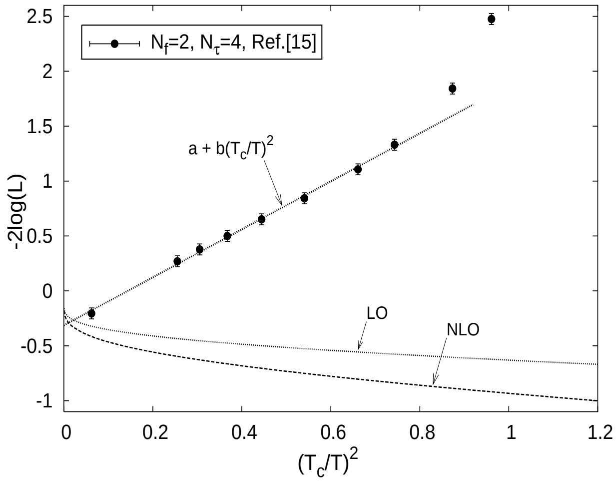

Figure 3: The logarithmic dependence of the renormalized Polyakov loop in unquenched QCD with two flavors versus the inverse temperature squared in units of the critical temperature. Lattice data from [15]. The fits use eq. (3.6) with a and b adjustable constants and data above $1.15\,T_c$. Purely perturbative LO and NLO results for $N_f = 2$ are shown for comparison.

4.2 Relation with zero temperature condensates

Although our determination is based on a static gauge, it is tempting to compare with the zero temperature condensate $g^2\langle A^2_{\mu,a}\rangle$ obtained in the Landau gauge in quenched QCD. There, one obtains from the gluon propagator $(2.4 \pm 0.6\,\mathrm{GeV})^2$ [21], from the symmetric three-gluon vertex $(3.6 \pm 1.2\,\mathrm{GeV})^2$ [21], and from the tail of the quark propagator $(2.1 \pm 0.1\,\mathrm{GeV})^2$ [22] and $(3.0 - 3.4\,\mathrm{GeV})^2$ [23]. At zero temperature all Lorentz components are sampled suggesting a conversion factor of 4 from $g^2\langle A^2_{\mu,a}\rangle$ to $g^2\langle A^2_{0,a}\rangle$, but according to [18], in the Landau gauge the total condensate scales as $D-1$, D being the Euclidean space dimension, suggesting instead a conversion factor of 3. Within the uncertainties of the lattice data as well as the theoretical ambiguities, the agreement is remarkable, as the two quenched results refer to different temperatures and gauges. Finite temperature results for the pressure in pure gluodynamics [24, 43] yield a value $(0.93(7)\,\mathrm{GeV})^2$ for $g^2\langle A^2_{0,a}\rangle^{\mathrm{NonPert}}$, indicating an overall coherent picture.[2]

4.3 Unquenched results

The renormalized Polyakov loop has also been computed in the unquenched case, using the technique described above, in ref. [15] for two flavor QCD. The lattice data are shown in figure 3, and they corresponds to $N_\tau = 4$. In this case, the data fall onto a straight line for temperatures $1.15\,T_c$ or above. Closer to the transition temperature the data start departing from the pattern (3.6), indicating the need of a richer description as the transition

[2] This value has been obtained from lattice data shown in figure 2 of ref. [24], and also from figure 1 of ref. [43], in the temperature region used in our fits.

is approached from above. A fit to the data above $1.15\,T_c$ using a^{NLO} yields

$$b = 2.99(12)\,, \quad g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}} = (0.86 \pm 0.02\,\mathrm{GeV})^2\,, \tag{4.12}$$

with $\chi^2/\mathrm{DOF} = 1.87$. We have used $T_c/\Lambda_{\overline{\mathrm{MS}}} = 0.77(9)$ with $T_c = 202(4)$ MeV [44] and $\Lambda_{\overline{\mathrm{MS}}} = 261(31)$ MeV [45]. The fit has been done with equal weight to all data points and the value of χ^2 quoted corresponds to a representative error ± 0.05 in $2\log L(T)$, which similar to that for the quenched case.

A fit with a and b as free parameters gives

$$a = -0.31(6)\,, \quad b = 2.19(13)\,, \quad g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}} = (0.73 \pm 0.03\,\mathrm{GeV})^2\,, \tag{4.13}$$

with $\chi^2/\mathrm{DOF} = 0.25$. As in the quenched case, the value of a is consistent with the perturbative value at high temperature

$$a^{\mathrm{NLO}} = -0.35(2) \qquad (T = 6\,T_c)\,. \tag{4.14}$$

The lattice data show a departure from the linear pattern for temperatures closer to the transition than $1.15\,T_c$. Such departure is not well described by adding new condensates of higher dimension and we have been unable to extract a condensate of dimension 4 from the data. We quote here the result of appending a term $c(T_c/T)^4$ in eq. (4.4). The fit of the data above $1.0\,T_c$ gives $b = 2.44(21)$ and $c = 1.07(19)$ with $\chi^2/\mathrm{DOF} = 12.8$. The coefficients b and c are highly correlated.

4.4 Further quenched lattice data

Alternative lattice determinations of the renormalized Polyakov loop in pure gluodynamics have been addressed more recently in [16]. These authors follow a different approach as compared to that in [14]. They use single Polyakov loops which are multiplicatively renormalized by extraction of the quark selfenergy. The latter is determined by isolating the cutoff dependent pieces by comparison of different lattice sizes at the same temperature. Unfortunately the results of both approaches differ qualitatively, specially for temperatures above $1.3\,T_c$. This is shown in figure 4 where the two lattice data sets are compared.

The origin of the discrepancy between the results obtained with the two approaches is presently not clear, although lattice artifacts, in particular finite lattice spacing effects, are not completely excluded in [16] as a possible explanation. (Of course, there is also the possibility that after closer scrutiny the two definitions used by the two groups correspond really to different renormalized operators.)

In our view the results in [14] would be the more reliable ones because the method used is technically simpler and amenable to tests. Indeed, the authors are able to verify that for small separations of the two Polyakov loops the standard zero temperature potential is very accurately reproduced as a function of r for all temperatures. This is achieved after a single (temperature dependent) global shift is made, to remove the quark selfenergies; this is the quantity $c(T)$ in eq. (4.1). The contact between the zero and finite temperature potentials is complete for all separations between zero and a T dependent radius $r(T)$ related to the Debye mass, thereby allowing a quite precise determination of the counterterm $c(T)$

JHEP01(2006)073

for each temperature. In addition, as noted above, the calculations are carried out for two different lattice sizes, $N_\tau = 4$ and $N_\tau = 8$ (and also $N_\tau = 16$ in [42]), and the results for the renormalized Polyakov show very small cutoff dependence, implying that the continuum limit has been reached.

The method in [16] is technically more difficult to implement (quoting the authors, "In practice, our method is not quite so trivial") since it requires comparing different lattice sizes at the same physical temperature. Also the subtraction of counterterms is more involved, since, using perturbation theory as guidance, the analogous of $c(T)$ is expressed as power series of T with coefficients to be fitted to the bare Polyakov loop data. On the other hand, from the point of view of the model proposed in the present work, we expect non perturbative corrections to be negligible at the highest temperatures considered in the two lattice calculations and only the data in [14] seem to be consistent with perturbation theory [12] at those temperatures.

The method in [16] renormalizes the logarithm of the bare Polyakov loop by using the scheme

$$-\log L^{\mathrm{bare}}(T) = f^{\mathrm{div}} N_\tau + f^{\mathrm{ren}} + f^{\mathrm{lat}} N_\tau^{-1} \tag{4.15}$$

where N_τ is the lattice temporal size, and so $N_\tau = \Lambda/T$, Λ being the inverse lattice spacing, i.e. the lattice cutoff. As said, the data in [16] deviate from those in [14], and in particular, do not follow the pattern (3.6) for $\log(L)$. Let us make a speculation assuming that either the removal of the cutoff dependent pieces has not been complete or that after removal of the those pieces, finite renormalization terms of the same type as the subtracted ones remain in the renormalized data of [16].[3] Specifically, let us assume that the data follow the pattern

$$-2\log L = a^{\mathrm{NLO}} + b\left(\frac{T_c}{T}\right)^2 + \delta a_{-1}\frac{T_c}{T} + \delta a + \delta a_1 \frac{T}{T_c}\,. \tag{4.16}$$

Actually, we find that the data above $1.3\,T_c$ can fairly well be accounted for by using this pattern. This is shown in figure 4. Remarkably, the central value of the slope b turns out to be close to that found previously with the other set of data. However, the best fit has large error bars due to the abundance of parameters available.

$$\begin{aligned} \delta a &= 1.8 \pm 1.8\,, & b &= 1.4 \pm 2.6\,, \\ \delta a_{-1} &= -1.0 \pm 3.8\,, & \delta a_1 &= -0.29 \pm 0.26\,, \end{aligned} \tag{4.17}$$

with $\chi^2/\mathrm{DOF} = 0.0349$.

Similar remarks apply to the fit

$$-2\log L = a + b\left(\frac{T_c}{T}\right)^2 + \delta a_{-1}\frac{T_c}{T} + \delta a + \delta a_1 \frac{T}{T_c}\,, \tag{4.18}$$

[3] Of course, one could also ask whether the result in [14] are not contaminated by finite cutoff effects too, and in particular, whether the linear pattern displayed in figure 4 is not just the consequence of a huge cutoff effect of the type Λ^2/T^2 instead of $\Lambda^2_{\mathrm{QCD}}/T^2$ as proposed in this work. This is unlikely, first because the values of the cutoff Λ in [14] are much larger than Λ_{QCD} and second, because the renormalized results are consistent for different lattice sizes, $N_\tau = 4$ and $N_\tau = 8$.

JHEP01(2006)073

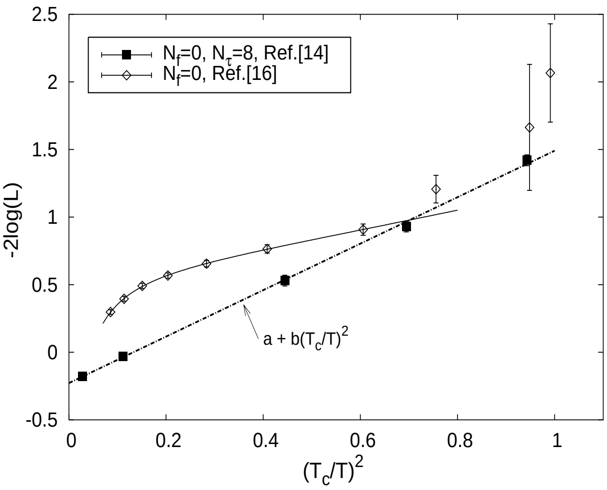

Figure 4: The logarithmic dependence of the renormalized Polyakov loop versus the inverse temperature squared in units of the critical temperature. Lattice data from [14, 16]. The fits use eq. (3.6) with a and b adjustable constants for the first set of data [14], and eq. (4.18) for the second one [16].

although in this case a and δa cannot be determined independently. This gives

$$\begin{aligned} a + \delta a &= 1.6 \pm 1.8\,, & b &= 1.3 \pm 2.6\,, \\ \delta a_{-1} &= -1.4 \pm 3.8\,, & \delta a_1 &= -0.28 \pm 0.26\,, \end{aligned} \tag{4.19}$$

with $\chi^2/\mathrm{DOF} = 0.0350$.

We find encouraging that the value of the condensate approximately agrees using the two different lattice data sets. Nevertheless, this speculation is not completely conclusive and an agreement between the results of both lattice groups would be needed before further consequences could be extracted.

5. Conclusions

There are two main results of our study. First, when suitably analyzed, the lattice data of the renormalized Polyakov loop above the deconfinement phase transition show unequivocally the existence of a non perturbative dimension 2 condensate. Such contributions have not been considered before but they are in fact dominant and allow to describe the data in [14] down to temperatures *as close to the transition as* $1.03\,T_c$ for pure gluodynamics and $1.15\,T_c$ for two flavors. Furthermore, the numerical value obtained from the Polyakov loop is quite consistent with the value of $g^2\langle A_{0,a}^2\rangle^{\mathrm{NonPert}}$ extracted from the pressure in gluodynamics.

We have suggested identifying this condensate with the BRST invariant dimension 2 gluon condensate. Our second finding is that, for pure gluodynamics, the numerical value of the condensate $\langle A_{0,a}^2\rangle^{\mathrm{NonPert}}$, defined in a static gauge and extracted from Polyakov loop data above the deconfinement transition, is remarkably close to the naive estimate $\langle A_{\mu,a}^2\rangle/4$, measured at zero temperature and in the Landau gauge. These results pose the theoretical challenge of establishing the connection outlined in this paper on a firmer ground. In this light the analogy between the zero temperature potential and the Polyakov loop noted in the introduction has been pushed forward in [46] by showing that the model in eq. (3.1) predicts a relation between the string tension and the slope of the Polyakov loop that is empirically satisfied.

The simple shape $L^2(T) = e^{-a-b(T_c/T)^2}$ yields $L \to 0$ as $T \to 0$, but does not describe the deconfinement phase transition. The closest analogy to such transition would be near the inflexion point of $L^2(T)$, which takes place at a temperature $T_i = (2b/3)^{1/2} T_c$. This T_i would agree with T_c for a universal geometrical value $b = 3/2$, which not far from the values obtained in this work from quenched QCD lattice data. Nevertheless, this approximate coincidence can only be taken as an estimate since the concrete value of the inflexion point depends on whether $L^2(T)$ or $L(T)$ is used, for instance. It is noteworthy that the same shape can also be obtained within the instanton approach at finite temperature along the lines of [47]. The relation between instantons and dimension 2 gluon condensates at zero temperature was suggested in [48] and fruitfully exploited in recent lattice simulations to extract, via cooling techniques, the infrared behavior of the running coupling constant [49]. In this regard, it might be rather interesting to isolate the purely non perturbative instantonic contributions on the lattice and determine whether, after cooling, the shape $e^{-(a+b(T_c/T)^2)/2}$ extends also below the phase transition.

Acknowledgments

We thank K. Kajantie for useful correspondence. This work is supported in part by funds provided by the Spanish DGI and FEDER funds with grant no. FIS2005-00810, Junta de Andalucía grant No. FQM-225 and EU RTN Contract CT2002-0311 (EURIDICE).

References

[1] J. Kuti, J. Polonyi and K. Szlachanyi, *Monte Carlo study of* SU(2) *gauge theory at finite temperature*, *Phys. Lett.* **B 98** (1981) 199.

[2] L.D. McLerran and B. Svetitsky, *Quark liberation at high temperature: a Monte Carlo study of* SU(2) *gauge theory*, *Phys. Rev.* **D 24** (1981) 450.

[3] G. 't Hooft, *On the phase transition towards permanent quark confinement*, *Nucl. Phys.* **B 138** (1978) 1.

[4] A.M. Polyakov, *Thermal properties of gauge fields and quark liberation*, *Phys. Lett.* **B 72** (1978) 477.

[5] L. Susskind, *Lattice models of quark confinement at high temperature*, *Phys. Rev.* **D 20** (1979) 2610.

[6] R.D. Pisarski, *Quark-gluon plasma as a condensate of* SU(3) *Wilson lines*, *Phys. Rev.* **D 62** (2000) 111501 [hep-ph/0006205].

[7] R.D. Pisarski, *Notes on the deconfining phase transition*, hep-ph/0203271.

[8] A.M. Polyakov, *Gauge fields as rings of glue*, *Nucl. Phys.* **B 164** (1980) 171.

[9] I.Y. Aref'eva, *Quantum contour field equations*, *Phys. Lett.* **B 93** (1980) 347.

[10] V.S. Dotsenko and S.N. Vergeles, *Renormalizability of phase factors in the nonabelian gauge theory*, *Nucl. Phys.* **B 169** (1980) 527.

[11] J.-L. Gervais and A. Neveu, *The slope of the leading Regge trajectory in quantum chromodynamics*, *Nucl. Phys.* **B 163** (1980) 189.

[12] E. Gava and R. Jengo, *Perturbative evaluation of the thermal Wilson loop*, *Phys. Lett.* **B 105** (1981) 285.

[13] A.D. Linde, *Infrared problem in thermodynamics of the Yang-Mills gas*, *Phys. Lett.* **B 96** (1980) 289.

[14] O. Kaczmarek, F. Karsch, P. Petreczky and F. Zantow, *Heavy quark anti-quark free energy and the renormalized Polyakov loop*, *Phys. Lett.* **B 543** (2002) 41 [hep-lat/0207002].

[15] O. Kaczmarek and F. Zantow, *Static quark anti-quark interactions in zero and finite temperature QCD, I. Heavy quark free energies, running coupling and quarkonium binding*, *Phys. Rev.* **D 71** (2005) 114510 [hep-lat/0503017].

[16] A. Dumitru, Y. Hatta, J. Lenaghan, K. Orginos and R.D. Pisarski, *Deconfining phase transition as a matrix model of renormalized Polyakov loops*, *Phys. Rev.* **D 70** (2004) 034511 [hep-th/0311223].

[17] S.-z. Huang and M. Lissia, *The relevant scale parameter in the high temperature phase of QCD*, *Nucl. Phys.* **B 438** (1995) 54 [hep-ph/9411293].

[18] M.J. Lavelle and M. Schaden, *Propagators and condensates in QCD*, *Phys. Lett.* **B 208** (1988) 297.

[19] K.G. Chetyrkin, S. Narison and V.I. Zakharov, *Short-distance tachyonic gluon mass and $1/Q^2$ corrections*, *Nucl. Phys.* **B 550** (1999) 353 [hep-ph/9811275].

[20] K.-I. Kondo, *Vacuum condensate of mass dimension 2 as the origin of mass gap and quark confinement*, *Phys. Lett.* **B 514** (2001) 335 [hep-th/0105299].

[21] P. Boucaud et al., *Testing Landau gauge ope on the lattice with a $\langle A^2 \rangle$ condensate*, *Phys. Rev.* **D 63** (2001) 114003 [hep-ph/0101302].

[22] E. Ruiz Arriola, P.O. Bowman and W. Broniowski, *Landau-gauge condensates from the quark propagator on the lattice*, *Phys. Rev.* **D 70** (2004) 097505 [hep-ph/0408309].

[23] P. Boucaud et al., *Artefacts and $\langle A^2 \rangle$ power corrections: revisiting the MOM $Z_\psi(p^2)$ and Z_V*, hep-lat/0504017.

[24] K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, *How to resum long-distance contributions to the QCD pressure?*, *Phys. Rev. Lett.* **86** (2001) 10 [hep-ph/0007109].

[25] S. Nadkarni, *Dimensional reduction in hot QCD*, *Phys. Rev.* **D 27** (1983) 917.

[26] K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, *Four-loop vacuum energy density of the* $\mathrm{SU}(N_c)$ *+ adjoint Higgs theory*, *JHEP* **04** (2003) 036 [hep-ph/0304048].

[27] P.H. Ginsparg, *First order and second order phase transitions in gauge theories at finite temperature*, *Nucl. Phys.* **B 170** (1980) 388.

JHEP01(2006)073

[28] T. Appelquist and R.D. Pisarski, *High-temperature Yang-Mills theories and three-dimensional quantum chromodynamics*, *Phys. Rev.* **D 23** (1981) 2305.

[29] E. Braaten and A. Nieto, *Free energy of QCD at high temperature*, *Phys. Rev.* **D 53** (1996) 3421 [hep-ph/9510408].

[30] M.E. Shaposhnikov, *Finite temperature effective theories*, hep-ph/9610247; *Effective theories and fundamental interactions*, Erice 1996, pp. 360–383.

[31] S. Chapman, *A new dimensionally reduced effective action for QCD at high temperature*, *Phys. Rev.* **D 50** (1994) 5308 [hep-ph/9407313].

[32] E. Megías, E. Ruiz Arriola and L.L. Salcedo, *The thermal heat kernel expansion and the one-loop effective action of QCD at finite temperature*, *Phys. Rev.* **D 69** (2004) 116003 [hep-ph/0312133].

[33] K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, *The pressure of hot QCD up to* $g^6 \ln(1/g)$, *Phys. Rev.* **D 67** (2003) 105008 [hep-ph/0211321].

[34] J. Engels, J. Fingberg and M. Weber, *Polyakov loop distributions near deconfinement in* SU(2) *lattice gauge theory*, *Z. Physik* **C 41** (1988) 513.

[35] M.A. Shifman, *Wilson loop in vacuum fields*, *Nucl. Phys.* **B 173** (1980) 13.

[36] K.-I. Kondo and T. Imai, *A confining string theory derivable from Yang-Mills theory due to a novel vacuum condensate*, hep-th/0206173.

[37] G.S. Bali, *QCD forces and heavy quark bound states*, *Phys. Rept.* **343** (2001) 1 [hep-ph/0001312].

[38] N. Brambilla, A. Pineda, J. Soto and A. Vairo, *The infrared behaviour of the static potential in perturbative QCD*, *Phys. Rev.* **D 60** (1999) 091502 [hep-ph/9903355].

[39] Y. Sumino, *Perturbative QCD potential and string tension*, hep-ph/0410013.

[40] E. Megías, E. Ruiz Arriola and L.L. Salcedo, *Polyakov loop in chiral quark models at finite temperature*, hep-ph/0412308.

[41] B. Beinlich, F. Karsch, E. Laermann and A. Peikert, *String tension and thermodynamics with tree level and tadpole improved actions*, *Eur. Phys. J.* **C 6** (1999) 133 [hep-lat/9707023].

[42] F. Zantow, *On the renormalization of the Polyakov loop*, hep-lat/0301014.

[43] K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, *Four-loop logarithms in 3d gauge + Higgs theory*, *Nucl. Phys.* **119** *(Proc. Suppl.)* (2003) 577 [hep-lat/0209072].

[44] F. Karsch, E. Laermann and A. Peikert, *The pressure in 2, 2+1 and 3 flavour QCD*, *Phys. Lett.* **B 478** (2000) 447 [hep-lat/0002003].

[45] M. Gockeler et al., *A determination of the* λ *parameter from full lattice QCD*, hep-ph/0502212.

[46] E. Megías, E. Ruiz Arriola and L.L. Salcedo, *Non-perturbative contribution to the Polyakov loop above the deconfinement phase transition*, hep-ph/0510114.

[47] E. Gava, R. Jengo and C. Omero, *On the instanton effect in the finite temperature Yang-Mills theory and in the nonlinear sigma model*, *Nucl. Phys.* **B 200** (1982) 107.

[48] M. Hutter, *Instantons in QCD: theory and application of the instanton liquid model*, hep-ph/0107098.

JHEP01(2006)073

[49] P. Boucaud et al., *Evidences for instantons effects in Landau lattice Green functions*, *Phys. Rev.* **D 70** (2004) 114503 [hep-ph/0312332].